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Optimal Search Theory and Optical Artificial Satellite Searches

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FOR THE COMMANDER

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OPTIMAL SEARCH THEORY AND OPTICAL ARTIFICIAL SATELLITE SEARCHES

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ABSTRACT

This Report discusses the along orbit search, by optical means, for an artificial satellite. In particular the attempt is made to couch the search in the existing scenario of optimal search theory. This cannot be done for existing and envisaged searches. The reasons for this are explored and some new concepts of optimality are discussed for real searches. The point is made that both hardware and software adjustments would be necessary in order to reconfigure optical searches for artificial satellites so that search theory can be utilized.

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I. INTRODUCTION

The theory of search, as a branch of operations research, had its beginnings during World War II. This start was made by Bernard Koopman and his colleagues for the United States Navy on problems related to anti-submarine warfare. Since then there has been considerable progress in the theoretical development of searches for non-moving targets.* Some progress has been made when the target is allowed to move in a conditionally deterministic fashion. Lawrence D. Stone has summarized much of the field in his book, "Theory of Optimal Search" published in 1975 by Academic Press. This Report is an introduction to this field of mathematics. I closely follow Stone's notation. This is to enable the interested reader to make a smooth transition to the literature should he care to pursue the subject further.

This Report is not a re-writing of Stone's book. Although I follow him in basic definitions and notation, I've chosen to illuminate the concepts within the framework of an along orbit search for an artificial satellite by optical sensors. As the Ground-Based Electro-Optical Deep Space Surveillance (GEODSS) system comes on-line, optical searches for artificial satellites will become more frequent. An along orbit search is a particularly simple search that can illustrate how optical searches for artificial satellites fit into the existing theoretical framework.

Actually such searches don't fit within the existing framework. A minor problem, for rapid searches on slow moving satellites, is the fact that the target is moving. When the search is executed slowly or the satellite is

* I deliberately avoid using the adjective stationary because of the special meaning it has in the artificial satellite context.

moving quickly, this becomes a major problem. The principal reason that the mathematical superstructure of optimal search theory is superfluous is that the optimization problem has been co-opted by the basic design of existing and currently contemplated searches. I will consider some of the reasons for this. This Report can serve as a stimulus for re-thinking why we search the way we do and whether or not we should modify it to obtain the benefits of the existing theory.

Given the existence of Stone's book I won't reference any other literature herein. All results specifically pertinent to the along-orbit optical search for an artificial satellite are original.

II. BASIC IDEAS

First we introduce the concept of a discrete search space. The target is in one of a set J of cells that are taken to be a (possibly infinite) subset of the positive integers. We can obtain a concrete realization of this in the along orbit optical search problem by imagining that the sensor has a field-of-view of θ radians and that our search lies along the artificial satellite's orbit. The maximum number of cells is $2\pi/\theta$. Geometrical constraints would limit this further. Also most along orbit searches are based on the assumption that the satellite will be "early" by a maximum amount and "late" by a maximum amount. This too would limit the maximum number of cells.

The second concept is that of an a priori target distribution. By this we mean the probability distribution over the search space which summarizes our knowledge of where the target was likely to be when the search commenced. We symbolize the target distribution by $p(j)$ where

$$p(j)\varepsilon[0,1]\,\forall j\varepsilon J \text{ and } \sum_{j\varepsilon J} p(j)\leq 1$$

In general we allow the target distribution to be defective. In the along orbit problem we might have to allow for a maneuver of the satellite which changed its orbital plane. If our search space consisted of only a few cells, then the satellite might not appear in any of them.

When we search a cell $j\varepsilon J$ we expend a certain amount of effort measured by $z\varepsilon[0,\infty)$. The cost of this effort is given by the cost function $c(j,z)$. Cost may be measured in time or money while effort might represent area searched or the duration of time spent in a cell. In the along orbit problem effort and cost might be both measured in time. The effort would be simply the amount of time devoted to searching cell j. The cost would be this time plus the amount of time necessary to move the telescope to cell j from its last location. Such a cost function is realistic and does <u>not</u> fit into the above framework. If the amount of time spent searching in a cell was much longer than the amount of time required to move to a cell, then we might ignore the non-local nature of the cost function. We could for instance, use a cost function of the form $c(j,z) - c_j + z$ where c_j is the average time to move to cell j from any other cell in J. At this early stage we've been forced to make two modifications of real along orbit searches in an attempt to fit into the formalism; real satellites are moving not fixed and real cost functions are non-local.

When we expend effort z in cell j we spend cost $c(j,z)$. In return for this we increase (or at least not decrease) the probability of detection. We define a <u>detection function</u> $b(j,z)$ that measures this. Specifically $b(j,z)$ is the conditional probability that after expending effort z in cell j we will detect the target given that it is in cell j. Note that this definition of the detection function assumes that the conditional probability of detection depends only on the total amount of effort expended and not on the way that the effort was expended. An obvious limitation we must impose on the cost function $c(j,z)$ is

$$c(j,z) \geq 0 \;\; \forall \; j \in J, \quad z \in [0,\infty)$$

Clearly we must demand that the detection function satisfy

$$b(j,z) \quad \varepsilon[0,1] \quad \forall j \varepsilon J, \; z \varepsilon [0,\infty)$$

There is a special class of detection functions known as <u>regular</u> detection functions that are important in the theoretical framework. The detection function $b(j,z)$ is regular if

(i) $b(j,0) = 0$ and

(ii) $\frac{\partial b(j,z)}{\partial z}$ is continuous, positive, and strictly decreasing $\forall j \varepsilon J$

The first requirement just says that we must expend effort if we are to find the target. The second requirement implies that $b(j,z)$ is strictly concave and that the <u>rate of return</u> function $\rho(j,z) = p(j)\frac{\partial b(j,z)}{\partial z}$ diminishes with increasing effort. The rate of return function tells us which cell yields the largest increase in probability of detection for a given, small, increment of effort. A search that always places the next increment of effort in the cell with the highest rate of return is called <u>locally optimal</u>.

The detection function $b(j,z)$ is something that we could model for a particular optical sensor. Presumably it would require only minor approximations (or perhaps no approximations) for such a detection function to be regular. As far as I am aware this has not been done. It would turn out to be superfluous for presently conceived searches though because current hardware constraints are such that one would not change the amount of effort allocated to a cell during a search. This ab initio fixing of the effort allocated to a cell (in fact to be the same constant amount for each cell) co-opts the optimization problem completely. We'll see this more clearly soon. Moreover, even if this were not the case, many types of artificial satellite searches have designs which prohibit the design of an optimal search in the sense discussed below. Searches that define J or searches that are designed to be leakproof can remove the essential degree of freedom necessary for the successful, meaningful application of search theory.

So far we have introduced the search space J of cells, the a priori probability distribution over the search space p(j), the cost of expending effort z in cell j, c(j,z), and the conditional probability of detecting the target in cell j after expending effort z there, b(j,z). Finally we introduce the concept of an <u>allocation</u> of effort over J, f(j). If we define the set F(J) by

$$F = F(J) = \text{Set of all non-negative functions } f \text{ defined on } J$$

then we can formulate the basic search problem.

Suppose we have an allocation of effort f(j) for each cell in the search space. Then the total cost C[f] of this allocation is

$$C[f] = \sum_{j \in J} c(j,f(j))$$

The total probability of detecting the target with this allocation is P[f],

$$P[f] = \sum_{j \in J} p(j)b(j,f(j))$$

Suppose that the maximum cost is K. The basic search problem is to find an allocation of effort $f^* \epsilon F$ such that

$$C[f^*] \leq K$$

and

$$P[f^*] = \max \left\{ P[f] : f \epsilon F \text{ and } C[f] \leq K \right\}$$

If such an allocation exists then it is said to be <u>optimal</u> for cost K.

Now we can clearly see the critical element of the above mentioned difficulty. We ignore the fact that the theory is concerned with fixed targets. We ignore the fact that the theory is capable of dealing only with local cost functions. We cannot, however, ignore the fact that operationally we ab initio determine that f(j) is a constant such that C[f] = K. This stricture removes the free element of the optimization problem. To fit optical artificial satellite searches within the theoretical framework we can not ab initio declare that f(j) is a constant. In order to free the allocation of effort requires a re-design of some hardware and software as well as careful examination of the current demands on artificial satellite searches (eg leakproofness).

After we introduce one more concept we can turn to the results of optimal search theory. The final concept is that of a <u>search plan</u>. A search plan $\phi(j,t)$ is a function defined on $j \epsilon J$, $t \epsilon [0,\infty)$ which tells us how much effort has been expended in cell j by time t. If M(t) gives the total effort available by time t then

$$\sum_{j \epsilon J} \phi(j,t) = M(t)$$

Let $\Phi(M)$ be the class of search plans satisfying the above. Then we say that a search plan $\phi^* \varepsilon \Phi(M)$ is uniformly optimal within $\Phi(M)$ if

$$P[\phi^*(j,t)] = \max \left| P[\phi(j,t)] : \phi \varepsilon \Phi(M) \right| \quad \forall\ t \geq 0$$

In other words such a search plan ϕ^* maximizes the probability of detection at every instant. Under certain conditions one can show that the locally optimal plan, the uniformly optimal plan, and the plan that minimizes the mean time to detection are identical.

III. THEORY

Above the basic concepts of search theory in a discrete search space have been introduced. In this section we shall see how optimal search plans can be found and computed. No proofs are provided; they are in Stone's book. Also, as I feel that it's more important to convey the sense of what can be demonstrated, rather than the details, I've taken some liberties in relaxing the wording of some of the results. Again see Stone's book for details.

Definition: An allocation of effort $f^* \epsilon F(J)$ is optimal for cost K if

$$C[f^*] \leq K \text{ and } P[f^*] = \max \left| P[f]:f \epsilon F(J) \text{ and } C[f] \leq K \right|$$

Theorem: Suppose there exists a $\lambda \epsilon [0,\infty)$ and an allocation $f^*_\lambda \epsilon F(J)$ such that $C[f^*_\lambda] < \infty$ and

$$P[f^*_\lambda] - \lambda C[f^*_\lambda] \geq P[f] - \lambda C[f] \text{ for } f \epsilon F(J) \ni C[f] < \infty$$

then

$$P[f^*_\lambda] = \max \left| P[f]:f \epsilon F(J) \text{ and } C[f] \leq C[f^*_\lambda] \right|$$

This theorem tells us when an allocation f^*_λ is optimal for a given cost $C[f^*_\lambda]$. The introduction of the Lagrange multiplier λ converts a constrained optimization problem for the functional $P[f]$ into an unconstrained optimization problem for $P[f]-\lambda C[f]$. This device suggests the introduction of the point-wise Lagrangian

$$\ell(j,\lambda,z) = p(j)b(j,z)-\lambda c(j,z) \text{ for } \forall\, j \epsilon J;\ \lambda,z \epsilon [0,\infty)$$

Note that

$$\sum_{j\varepsilon J} \ell(j,\lambda,f) = P[f]-\lambda C[f]$$

<u>Theorem</u>: Suppose there exists a $\lambda\varepsilon[0,\infty)$ and an allocation $f^*_\lambda\varepsilon F(J)$ such that $C[f^*_\lambda]<\infty$ and

$$\ell(j,\lambda,f^*_\lambda(J)) = \max \left|\ell(j,\lambda,z):z\varepsilon[0,\infty)\right| \quad \forall j\varepsilon J$$

then

$$P[f^*_\lambda] = \max \left|P[f]:f\varepsilon F(J) \text{ and } C[f]\leq C[f^*_\lambda]\right|$$

Note that in this simple case once j and λ are fixed $\ell(j,\lambda,z)$ is a function of one variable and the apparatus of ordinary differential calculus may be used to find its maximum. The above theorems give sufficient conditions that a particular allocation satisfies the constrained optimization problem. To go further we must assume a bit more about the detection function and the cost function. Moreover we say that whenever (λ,f^*) are such that $f^*\varepsilon F(J)$ and $\lambda\varepsilon[0,\infty)$ and

$$\ell(j,\lambda,f^*(j)) = \max \left|\ell(j,\lambda,z):z\varepsilon[0,\infty]\right| \quad \forall j\varepsilon J$$

that (λ,f^*) maximizes the pointwise Lagrangian.

<u>Theorem</u>: Let $b(j,z)$ be a concave function of z and $c(j,z)$ be a convex function of z. Let $f^*\varepsilon F(J)$ and $C[f^*]\varepsilon(0,K)$. Then a necessary and sufficient condition for f^* to be optimal for cost $C[f^*]$ is that there exists a $\lambda\varepsilon[0,\infty)$ such that (λ,f^*) maximizes the pointwise Lagrangian.

Corollary: Let $\partial b(j,z)/\partial z$ be a decreasing and continuous function of z and let $\partial c(j,z)/\partial z$ be an increasing and continuous function of $z \; \forall j \varepsilon J$. If $f^* \varepsilon F(J)$ is optimal for cost $C[f^*] \varepsilon (0,K)$ then there exists a $\lambda \geq 0$ such that

$$p(j) \frac{\partial b(j,f^*(j))}{\partial z} - \lambda \frac{\partial c(j,f^*(j))}{\partial z} \begin{cases} = 0 \text{ if } f^*(j) > 0 \\ \leq 0 \text{ if } f^*(j) = 0 \end{cases}$$

To complete the elementary part of the theory we need the inverse of the rate of return function $\rho(j,z) = p(j)\, \partial b(j,z)/\partial z$, viz.

$$\rho^{-1}(j,x) = \begin{cases} \text{inverse of } \rho(j,z) \text{ evaluated at } z=x \text{ for } x\varepsilon(0,\rho(j,0)] \\ 0 \qquad \text{for } x > \rho(j,0) \end{cases},$$

We also need the function

$$U(x) = \sum_{j\varepsilon J} \rho^{-1}(j,x)$$

and its inverse U^{-1}.

Theorem: If $c(j,z)=z \; \forall z\varepsilon[0,\infty)$ and $j\varepsilon J$ and $b(j,z)$ is a regular detection function then for a fixed cost $K>0$ the allocation

$$f^*_\lambda = \rho^{-1}\,(j,\lambda) \; \forall j\varepsilon J$$

where $\lambda = U^{-1}(K)$ is optimal for cost K and $C[f^*_\lambda] = K$.

Theorem: Under the above conditions the search plan

$$\phi^*\,(j,t) = \rho^{-1}\left(j,U^{-1}(M(t))\right) \; \forall j\varepsilon J,\; t>0$$

is uniformly optimal for cumulative effort $M(t)$ in $\Phi(M)$.

Finally define the mean time to find the target when using search plan ϕ, $\mu(\phi)$. We can write

$$\mu(\phi) = \int_0^\infty (1-P[\phi(j,t)])dt$$

Theorem: Let the search plan ϕ^* be uniformly optimal in $\Phi(M)$. Then

$\forall\phi\epsilon\Phi(M)$ $\qquad\qquad\qquad \mu(\phi^*)\leq\mu(\phi)$.

The above theorems completely solve the problem of constructing optimal search plans for regular detection functions and cost functions proportional to the effort expended. In order to give some life to theory, consider the along orbit search again. Let there be J cells, let the target distribution be p(j), and suppose that the detection function $b(j,z) = 1-\exp(-a_j z)$, $a_j\geq 0$. Note that this detection function is regular. Fix the total cost at K and let the cost function be $c(j,z)=z$. We will construct an algorithm for finding the optimal allocation of effort $f(j)=z_j$.

The total probability of detection for allocation f is

$$P[f] = \sum_{j=1}^{J} p(j)b(j,f(j)) = \sum_{j=1}^{J} p(j)(1-\exp(a_j z_j))$$

The total cost for this allocation is

$$C[f] = \sum_{j=1}^{J} c(j,f(j)) = \sum_{j=1}^{J} z_j$$

We seek to maximize P[f] subject to the constraint C[f]=K. We do this by introducing the Lagrange multiplier λ and minimizing P[f]-λC[f] with respect to $\{z_j\}$. We find that $\partial(P[f]-\lambda C[f])/\partial z_j=0$ implies

$$p(j)a_j\exp(-a_jz_j) = \lambda \quad \forall j\varepsilon[1,J]$$

Hence

$$z_j = (1/a_j)\ \ell n\ (p(j)a_j/\lambda)$$

Notice that the implicit equation for z_j is just $\rho(j,z_j) = \lambda$ and that the solution is just $z_j = \rho^{-1}(j,\lambda)$. Of course, we can only consider non-negative effort so if $p(j)a_j/\lambda<1, z_j=0$. We determine λ from the total cost constraint,

$$\sum_{j=1}^{J} z_j = K = \sum_{j=1}^{J} (1/a_j)\ \ell n\ (p(j)a_j/\lambda) = U(\lambda)$$

where only those terms are included whose argument of the natural logarithm are ≥ 1. This completes the solution of the problem. To see even more clearly how the search evolves suppose that the cells are labeled such that

$$a_1p(1) \geq a_2p(2)\ .\ .\ .\ .\ .\ \geq a_Jp(J)$$

If the total cost K is small only cell #1 will be searched for $p(j)a_j$ will be less than λ for $j>1$. As the total cost increases additional cells will be searched with the effort partitioned between them according to the above rules.

IV. ADDITIONAL TOPICS

The above theoretical development concerned optimal search plans over a discrete search space. The amount of effort expended was infinitely divisible though. A more realistic approach would be to quantify the amount of effort expended in each cell. All expenditures of effort would then occur in discrete multiples of the minimum. In optical searches the minimum amount of effort is the time to form an image with the sensor. Clearly it makes no sense to allocate an amount of effort (measured in time) that is less than the single image integration time.

Such searches are known as search with discrete effort. A logical measure of the cost is the amount of time needed to perform a single look (form an image) in a cell. The quantity subject to variation in the optimization problem is the distribution of the number of looks per cell. This is analogous to the allocation of effort discussed above. One can also carry over the notion of a locally optimal search. Such a search plan looks in that cell that yields the highest value for the quantity (increment in probability)/(increment in cost). If this ratio decreases with the number of looks (in every cell of the search space), then the locally optimal plan minimizes the mean time to find the target. If in addition we do measure cost by the number of looks per cell, then the locally optimal plan is also uniformly optimal, i.e. it maximizes the probability of detection for any number of looks >0.

Other topics treated by the theory include whereabouts searches, optimal search and stop, search in the presence of false targets, the approximation of optimal search plans, some small steps in solving the conditionally deterministic target motion problem, and Markovian target motion. The interested reader is referred to Stone's book.

V. THE ALONG ORBIT SEARCH

Within the framework of currently conceived searches there would appear to be little left to discuss. One can still pose interesting problems for along orbit searches depending on one's concept of optimality. In this Section I'll look at some of the simple problems an along orbit search presents.

A. Search Scenario

The search is constrained in the following fashion. The search space is a set of 2N+1, $N \geq 0$ (an integer) field-of-view θ. The fields are labeled by $n = -N, -N+1, \ldots, N$. All searches commence at $n = 0$, the nominal position for the artificial satellite. The 2N+1 fields lie along the satellite's orbital plane. The target distribution p(n) is specified. The restrictions

$$1 \geq p(0) \geq p(n) \geq 0 \quad \forall n \varepsilon [-N,N]$$

might be imposed with little loss of generality. In addition one might impose a symmetry constraint $p(n) = p(-n) \, \forall \, n \varepsilon [-N,N]$.

Each cell is searched with the same amount of effort. Moreover we assume that the detection function is homogeneous, $b(n,z) = b(m,z)$ $\forall \, n,m \varepsilon [-N,N]$. We define a search plan as a set of 2N+1 integers $n_0, n_1, n_2, \ldots n_{2N}$ drawn from [-N,N] subject to the constraints that $n_0 = 0$ and that there is no repetition. There are (2N)! different search plans of which half are the reflection of the other half. The cost function we develop below after we construct a model for the telescope's motion. Then we can pose several questions: 1) Which search plan takes the least time to complete an examination of all 2N+1 cells?, 2) Which search plan has the highest average probability of finding the target as a function of time? 3) Which search plan has the highest aggregate probability of detection for all times during the search? (The aggregate probability of detection is

$$P(t) = \sum_{n \in M(t)} p(n)$$

where M(t) is the set of cells searched by time t.), and 4) Do any of these matter in real world along orbit optical artificial satellite searches?

B. A Model for the Telescope

In order to define a realistic cost function we need a model for the telescope motion. For this purpose I assume that when the telescope starts from rest that it is capable of a maximum constant acceleration α for a maximum time τ. Hence the maximum angular speed of the telescope is $\Omega = \alpha\tau$. The telescope can move at the rate of Ω for an arbitrary length of time. When the telescope decelerates, it does so at a constant deceleration δ ($\delta > 0$) until it comes to rest or a specified angular speed $\omega, |\omega| \leq \Omega$.

Suppose that the problem is to move the telescope, initially at rest at $\phi = \phi_i$ to some other position $\phi = \phi_f > \phi_i$ where it will again be at rest. This process can occur in a maximum of three phases. During phase I the telescope accelerates at the rate of α for a total time $t_a \leq \tau$. During phase II the telescope moves at the constant angular speed $\omega_a = \alpha t_a$ for a time t_c. During phase III the telescope decelerates at the rate δ until it reaches a stop at $\phi = \phi_f$. This takes an addition time t_d. The total time of the move is $T = t_a + t_c + t_d$. Question: What combination of t_a, t_c and t_d minimizes T?

To solve this problem, we need equations of motion for the telescope. They are

Phase I: $t \epsilon [0, t_a]$

$$\phi = \alpha t^2/2 + \phi_i , \quad \omega = \alpha t$$

Phase II: $t - t_a \ \epsilon [0, t_c]$ $\quad \phi_a = \alpha t_a^2/2 + \phi_i , \quad \omega_a = \alpha t_a$

$$\phi = \omega_a (t - t_a) + \phi_a , \quad \omega = \omega_a$$

Phase III: $t - (t_a + t_c) \ \epsilon [0, t_d]$ $\quad \phi_c = \omega_a t_c + \phi_a , \quad \omega_c = \omega_a$

$$\phi = -\delta[t - (t_a + t_c)]^2/2 + \omega_c[t - (t_a + t_c)] + \phi_c ,$$

$$\omega = -\delta [t - (t_a + t_c)] + \omega_c$$

Set $t = t_a + t_c + t_d$ in the Phase III equations and insist that $\phi = \phi_f , \omega = 0$. One finds

$$t_d = \alpha t_a/\delta , \quad t_c = \frac{\Delta\phi}{\alpha t_a} - \frac{t_a}{2} (1 + \alpha/\delta)$$

where $\Delta\phi = \phi_f - \phi_i$. The total traverse time T is

$$T = \frac{t_a}{2} (1 + \alpha/\delta) + \frac{\Delta\phi}{\alpha t_a}$$

Considered as a function of t_a, T has a single minimum when

$$t_a^2 = \frac{2\Delta\phi}{\alpha(1 + \alpha/_\delta)}$$

which implies that the coast time, t_c is zero. The total time to move an angular distance $\Delta\phi$ is

$$T_{min} = \left[\frac{2(\alpha+\delta)}{\alpha\delta} \, |\Delta\phi|\right]^{1/2}$$

The above assumes that $t_a < \tau$. Should the above value of t_a be greater than or equal to τ, then the single degree of freedom is removed and

$$T = \frac{\Omega(\alpha+\delta)}{2\alpha\delta} \left[1 + \frac{2\alpha\delta|\Delta\phi|}{\Omega^2(\alpha+\delta)}\right]$$

Only for an extremely fast telescope or a very long along orbit search would t_a exceed τ.

C. Optimal Searches

The cost function we'll use is the time required to move from the last cell examined to the cell of interest plus the time spent examining the cell. The time per cell (= the effort expended) is a constant equal to t_{look}. Since the center of cell $n\varepsilon[-N,N]$ is at $\phi = n\theta$, for $t_a<\tau$ the cost function for the K'th cell is

$$\left[\frac{2(\alpha+\delta)\theta}{\alpha\delta} |n_K - n_{K-1}|\right]^{1/2} + t_{look}$$

Consider first a complete search (i.e. each cell is examined). Then the total time to complete the search is

$$(2N+1)t_{look} + \left[\frac{2(\alpha+\delta)\theta}{\alpha\delta}\right]^{1/2} \left\{ |n_1 - n_0|^{1/2} + |n_2 - n_1|^{1/2} + \ldots + |n_{2N} - n_{2N-1}|^{1/2} \right\}$$

Clearly when determining an optimal search only the sum of the square roots are important.

Conjecture: The complete search plan that has the shortest time to complete is the following (or its reflection): Pick a direction and move in it, one cell at a time, until the last cell on that side is reached. Next jump, in the other direction, one cell past the start and repeat the (new) unidirectional one cell at a time traverse. The time to complete is $2N+(N+1)^{1/2}-1$.

The along orbit searches actually used is an alternating one, eg. $n_0=0$, $n_1=+1$, $n_2=-1$, $n_3=+2$, $n_4=-2$, etc. The time to complete is

$$\sum_{n=1}^{2N} n^{1/2} \sim N^{3/2}$$

and therefore very long. This search does build up aggregate probability quickly though, especially for a sharply peaked, unimodal, symmetric distribution. If the arc of the orbit is long, then this probably isn't optimal in the sense of maximum aggregate probability. The reason is that since p(n) falls off rapidly with n but the time to move increases as the square root of n, for large enough of n it will be better to do cell n+1 after cell n and then jump to cell -n, do cell -(n+1), then jump to cell n+2, etc. Clearly one needs real numbers for α, δ, θ, and t_{look} to decide the question. The same is true for the highest average probability searches.

D. Does it matter?

Stone references, but does not deal with, the subset of the search literature concerned with searches along a line. I've briefly looked at it and it appears to be irrelevant. Finally, unless one has an extremely slow telescope or contemplates very long (in the sense of arc) along orbit searches, it is doubtful that considerations of optimality really matter. Note though that implicit in current along orbit searches is the assumption that $p(0) \sim \frac{1}{2}$, $p(\pm 1) \sim \frac{1}{4}$, or the satellite maneuvered.

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This report discusses the along orbit search, by optical means, for an artificial satellite. In particular the attempt is made to couch the search in the existing scenario of optimal search theory. This cannot be done for existing and envisaged searches. The reasons for this are explored and some new concepts of optimality are discussed for real searches. The point is made that both hardware and software adjustments would be necessary in order to reconfigure optical searches for artificial satellites so that search theory can be utilized.

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